The Optimal Graduated Minimum Wage and Social Welfare

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Abstract

This paper analyzes the effects of introducing a graduated minimum wage in a model with optimal income taxation in which a government seeks to maximize social welfare. It shows that the optimal graduated minimum wage increases social welfare by increasing the low-productivity workers' consumption and bringing it closer to the first-best. The paper also describes how the graduated minimum wage in a social welfare optimum depends on important economy characteristics such as the government's revenue needs and the numbers and productivities of the different types of workers.

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1 Introduction

It has long been recognized that in a competitive economy with optimal nonlinear income taxation and variable working hours, a *constant* minimum wage cannot improve social welfare (Allen, 1987; Guesnerie and Roberts, 1987). Recently, however, Danziger and Danziger (2015) have shown that, by slackening the high-productivity workers' incentive-compatibility constraint, a *graduated* minimum wage (that ties the minimum wage a firm must pay to the firm's size) can provide a strict Pareto improvement even in the presence of an optimal nonlinear income tax. Since that paper focussed on showing only that a Pareto improvement is feasible, it was ill-equipped to address questions about the optimal graduated minimum wage when the goal is to maximize social welfare.[1]

Our goal in this paper, therefore, is to explore the properties of the optimal graduated minimum wage. Indeed, it can be shown that when the goal is to maximize social welfare, the optimal graduated minimum wage need not provide a Pareto improvement over the allocation with only an optimal nonlinear income tax. Nevertheless, the fact that a graduated minimum wage *can* provide a strict Pareto improvement guarantees that even when it does not, it still must strictly increase social welfare compared to the allocation with only an optimal nonlinear income tax.

In order to explore the properties of the optimal graduated minimum wage we employ a modified Stiglitz (1982) framework with low- and high-productivity workers and optimal nonlinear income taxation. This framework allows us to address several interesting questions.

[1] For analyses of a constant minimum wage with variable working hours if the environment is not competitive or taxation not optimal, see Rebitzer and Taylor (1995), De Fraja (1999), Boadway and Cuff (2001), Bhashar et al. (2002), Blumkin and Sadka (2005), Strobl and Walsh (2007, 2011), Kaas and Madden (2008, 2010), Hungerbühler and Lehmann (2009), and Basu et al. (2010). If working hours are fixed and employment is rationed such that the involuntary unemployment induced by the minimum wage falls entirely on the low-productivity workers with the smallest surplus from working, then a constant minimum wage is optimal (Lee and Saez, 2012). See also Cahuc and Laroque (2014). The importance of cultural and institutional factors in the determination of a minimum wage is emphasized in Sobel (1999), Cahuc et al. (2001), Belot et al. (2007), Boeri and Burda (2009), Aghion et al. (2011), and Boeri (2012).

In particular, if the goal is to maximize social welfare, what effect does the introduction of a graduated minimum wage have on the low-productivity workers' consumption? And how does the graduated minimum wage in a social optimum depend on central characteristics of the economy such as the government's revenue needs and the numbers and productivities of the different types of workers?

We follow the optimal tax literature in assuming that income taxes can only depend on a worker's income and not be directly conditioned on wages or working hours.[2] We also assume that a graduated minimum wage can be enforced by a system of self-reporting and potential whistle blowing by disgruntled workers leading to inspections and heavy penalties for noncompliance. Indeed, it is easy for workers to know if they are paid too little as the intended minimum wage will be the smallest possible in the graduated minimum wage schedule.[3]

In our model, a graduated minimum wage ties the minimum wage a firm must pay to the employment of its eligible workers. Accordingly, it is a nonlinear function of the working hours of a firm's low-productivity workers that forces firms to choose between different combinations of minimum wage and corresponding working hours. The graduated minimum wage is thus analogous to a nonlinear income tax that forces workers to choose between different combinations of consumption and corresponding income.

A major finding is that a government that strives to maximize social welfare will always want to use a graduated minimum wage to increase the low-productivity workers' consump-

[2] Even though it would be preferable that taxes depend on wages and thus be a function of worker productivities, there seems to be a strong taboo against such type-based taxation.

[3] Workers also have an incentive to complain if employment is less than intended as the graduated minimum wage schedule would then require them to be paid more. Nevertheless, a graduated minimum wage may be more difficult to enforce than a constant minimum wage due to the need to keep track of the low-productivity workers' hours. However, in this respect the graduated minimum wage is not different from other government programs – such as the Work Opportunity Tax Credit program and the Welfare-to-Work Tax Credit program – that also require keeping track of hours worked. Furthermore, many countries (as well as thirteen U.S. states) have legislated minimum wages which depend on firm size or have a minimum wage that depends on the sector or occupation.

tion. We also obtain the intuitively appealing results that the optimal graduated minimum wage decreases with the government's revenue needs and the number of low-productivity workers, but increases with the number of high-productivity workers and the productivities of both types of workers.

2 The Model

The economy contains a continuum $n_1 > 0$ of low-productivity workers. Their utilities are given by $u(c_1) - h_1$, $u' > 0$ and $u'' < 0$, where c_1 and h_1 denote a low-productivity worker's consumption and working hours. The economy also contains a continuum $n_2 > 0$ of high-productivity workers whose utilities are given by $u(c_2) - h_2$, where c_2 and h_2 denote a high-productivity worker's consumption and working hours.[4]

In addition, there is a unit continuum of identical firms that produce a single consumption good whose price is normalized to unity. A firm's output is given by $af(\ell_1) + b\ell_2$, where ℓ_1 and ℓ_2 are the total hours of low- and high-productivity workers, respectively, hired by a firm; $a > 0$ and $b > 0$ their productivity levels; $f(0) = 0$, $f' > 0$, $f'' < 0$; and $af'(0) < b$. The production function therefore exhibits decreasing returns to scale, with the low-productivity workers having a decreasing marginal product which is always less than the high-productivity workers' marginal product.[5] Since there is a unit continuum of firms, in equilibrium labor-market clearing implies $\ell_1 = n_1 h_1$ and $\ell_2 = n_2 h_2$.

The government has a utilitarian social-welfare function

$$n_1 \left[u(c_1) - h_1\right] + n_2 \left[u(c_2) - h_2\right], \tag{1}$$

[4] The assumption that preferences are quasi-linear in working hours greatly facilitates the analysis of problems involving optimal taxation; see Boadway et al. (2000).

[5] Low-productivity workers must have a decreasing marginal product in order for a graduated minimum wage to be desirable. Our results would not change if high-productivity workers also have a decreasing marginal product.We assume that the low-productivity workers' production cannot be outsourced.

and the resource constraint of the economy is

$$af(\ell_1) + b\ell_2 - n_1c_1 - n_2c_2 = R, \tag{2}$$

where $R \geq 0$ is the government's exogenous revenue needs to finance public expenditures. The government determines a nonlinear income tax and possibly also a graduated minimum wage, and its goal is to maximize social welfare. Following the income-tax literature, we assume that the government can condition the income tax a worker pays only on her total income and not on her hourly wage.

2.1 The Benchmark Case: Social Welfare without a Graduated Minimum Wage

Suppose that wages are competitively determined so that the low-productivity workers' wage is $w_1 = af'(\ell_1)$ and the high-productivity workers' wage is $w_2 = b$. Also, in the benchmark case, suppose that the government enacts a nonlinear income tax but not a graduated minimum wage. Since the government can only distinguish workers based on their income, the income-tax scheme consists of consumption-income bundles offered to workers. The government can then differentiate between the two types of workers by designing the income tax so that each type of worker prefers the consumption-income bundle meant for her own type rather than for the other type (and making any other available bundle less attractive). This leads to the following incentive-compatibility constraints of the low- and high-productivity workers

$$u(\hat{c}_1) - \hat{h}_1 \geq u(\hat{c}_2) - \frac{\hat{w}_2\hat{h}_2}{\hat{w}_1}, \tag{3}$$

$$u(\hat{c}_2) - \hat{h}_2 \geq u(\hat{c}_1) - \frac{\hat{w}_1\hat{h}_1}{\hat{w}_2}, \tag{4}$$

where a circumflex is used to denote the optimal value of a variable. The last term on the right-hand-side of the constraints are the hours that one type of worker must work in order to earn the same income as the other type of worker.

The government seeks to maximize social welfare (1) by devising an income-tax function that determines the consumption-income bundles for the low- and high-productivity workers subject to the resource constraint (2) and the incentive-compatibility constraints (3) and (4).[6] Assuming an internal solution, it is straightforward to show that in a social-welfare optimum with only a nonlinear income tax, constraints (2) and (4) but not (3) are binding. Standard derivations establish that the low- and high-productivity workers' consumption satisfy

$$\begin{aligned} u'(\hat{c}_1) &= \frac{n_1 b + n_2 a f'(\hat{\ell}_1) + n_2 \hat{\ell}_1 a f''(\hat{\ell}_1)}{b[(n_1 + 2n_2) a f'(\hat{\ell}_1) - n_2 b + n_2 \hat{\ell}_1 a f''(\hat{\ell}_1)]}, \\ u'(\hat{c}_2) &= \frac{1}{b}. \end{aligned} \tag{5}$$

Put in words, for each type of worker the marginal utility of consumption is equal to the marginal social cost of producing that consumption. While the social cost of additional high-productivity worker consumption is exactly equal to their disutility from the additional work required to produce this additional consumption, the situation is different for low-productivity workers. Indeed, because additional low-productivity worker consumption tightens the incentive-compatibility constraint of high-productivity workers, the marginal social cost of additional low-productivity worker consumption exceeds the utility cost from the additional work required to produce this consumption.

2.2 Social Welfare with a Graduated Minimum Wage

Suppose that in addition to a nonlinear income tax, the government can enact a graduated minimum wage $m(\ell_1)$ that sets the minimum wage as a function of the total working hours of a firm's low-productivity workers. Thus, in addition to using the income tax to present workers with a choice between different consumption-income bundles, the government can

[6] Firms' profits, if any, are taxed away. Alternatively, workers own the firms equally with the income tax taking the distributed profits into account.

now also use a graduated minimum wage to present firms with a choice between different minimum wage-hours bundles. Accordingly, the wage for low-productivity workers is no longer competitively determined. However, the wage for high-productivity workers is still competitively determined and equal to b.

The incentive-compatibility constraints of the low- and high-productivity workers are modified to

$$u(c_1^*) - h_1^* \geq u(c_2^*) - \frac{w_2^* h_2^*}{m(\ell_1^*)}, \tag{6}$$

$$u(c_2^*) - h_2^* \geq u(c_1^*) - \frac{m(\ell_1^*) h_1^*}{w_2^*}, \tag{7}$$

where an asterisk is used to denote the optimal value of a variable.

The government, however, is constrained in its choice of the realized minimum wage, that is, the one actually paid by firms, $m(\ell_1^*)$. The reason is that if $m(\ell_1^*)$ is set too high, then firms may prefer some other point on the graduated minimum-wage schedule. Also, since, by definition, the minimum wage is the lowest wage in the economy, it must be the case that $m(\ell_1) \leq w_2$ for all values of ℓ_1. Therefore, the most attractive that the government can make ℓ_1^* relative to any alternative ℓ_1 is to set the graduated minimum wage at all points other than ℓ_1^* of ℓ_1 to be as prohibitive as possible, i.e., $m(\ell_1) = w_2$ for $\ell_1 \neq \ell_1^*$. Such a graduated minimum-wage schedule confronts a firm with two alternatives. The firm can either hire ℓ_1^* hours of low-productivity labor at wage $m(\ell_1^*)$, or the firm can choose to pay its low-productivity workers w_2 and be free to set their working hours as it desires. Therefore, the realized minimum wage, $m(\ell_1^*)$, must satisfy *the minimum-wage constraint,* that is, a firm must prefer to hire ℓ_1^* hours of low-productivity labor at a wage of $m(\ell_1^*)$, rather than choose a different ℓ_1 and pay the higher w_2.

However, since $af'(0) < b$ implies that $af(\ell_1) - w_2\ell_1 < 0$ for $\ell_1 > 0$, it follows that if a firms has to pay w_2 to the low-productivity workers, then its maximum profit will be zero and obtained at $\ell_1 = 0$. Therefore, accounting for the fact that the government would optimally set $m(\ell_1) = w_2$ for $\ell_1 \neq \ell_1^*$, the minimum-wage constraint reduces to

$$af(\ell_1^*) - m(\ell_1^*)\ell_1^* \geq 0. \tag{8}$$

The government's optimization problem is then to maximize social welfare (1) by designing an income-tax function that determines the consumption-income bundles for the low- and high-productivity workers, and a graduated minimum-wage schedule, $m(\ell_1)$. The allocation must satisfy the resource constraint (2), the modified incentive-compatibility constraints (6) and (7), and the minimum-wage constraint (8). Similarly to the case of only a nonlinear income tax, it can be shown that in any internal solution for a social-welfare optimum with both a nonlinear income tax and a graduated minimum wage only constraints (2), (7), and (8) are binding. The solution to the government's problem of maximizing welfare subject to these constraints implies that low- and high-productivity workers' consumptions are such that

$$\begin{aligned} u'(c_1^*) &= \frac{n_1 b + n_2 a f'(\ell_1^*)}{b[(n_1 + 2n_2)af'(\ell_1^*) - n_2 b]}, \\ u'(c_2^*) &= \frac{1}{b}. \end{aligned} \tag{9}$$

As in the benchmark case with only an optimal nonlinear income tax, the marginal social cost of low-productivity-worker consumption exceeds the utility loss from producing this additional consumption due to the tightening of the incentive-compatibility constraint on high-productivity workers. However, as we will show in the next section, the optimal graduated minimum wage reduces the marginal social cost of low-productivity-worker consumption relative to the scenario with only an optimal nonlinear income tax, thereby facilitating an improvement in social welfare.

3 Optimal Consumption with a Graduated Minimum Wage

Since the high-productivity workers' consumption is at the first-best level in a social-welfare optimum with only a nonlinear income tax and the social cost of their consumption ($= 1/b$) is not affected by the graduated minimum wage, their consumption in a social optimum remains unchanged at the first-best level when a graduated minimum wage is introduced.[7] We now show that the low-productivity workers' consumption is positively affected by the graduated minimum wage:

Proposition 1 *Low-productivity workers' consumption is higher with both an optimal graduated minimum wage and an optimal nonlinear income tax than with only an optimal nonlinear income tax; that is,* $c_1^* > \hat{c}_1$.

Proof. First, if $h_1^* = \hat{h}_1$, then (5) and (9) imply that

$$\begin{aligned}
& u'(\hat{c}_1) - u'(c_1^*) \\
= \ & \frac{n_1 b + n_2 a f'(\ell_1^*) + n_2 \ell_1^* a f''(\ell_1^*)}{b[(n_1 + 2n_2) a f'(\ell_1^*) - n_2 b + n_2 \ell_1^* a f''(\ell_1^*)]} - \frac{n_1 b + n_2 a f'(\ell_1^*)}{b[(n_1 + 2n_2) a f'(\ell_1^*) - n_2 b]} \\
= \ & \frac{n_2 \ell_1^* a f''(\ell_1^*)[n_1 a f'(\ell_1^*) - (n_1 + n_2) b]}{b[(n_1 + 2n_2) a f'(\ell_1^*) - n_2 b + n_2 \ell_1^* a f''(\ell_1^*)][(n_1 + 2n_2) a f'(\ell_1^*) - n_2 b]} \\
> \ & 0.
\end{aligned}$$

Accordingly, if $h_1^* = \hat{h}_1$, then $c_1^* > \hat{c}_1$.

Next, if $h_1^* < \hat{h}_1$, we note that the derivative of the marginal social cost of c_1^* (the right-hand side of (9)) with respect to h_1^* is

$$-\frac{n_1 (n_1 + n_2)^2 a f''(\ell_1^*)}{[(n_1 + 2n_2) a f'(\ell_1^*) - n_2 b + z]^2} > 0.$$

[7] Thus, the optimality of a zero marginal tax rate for the high-productivity workers in the standard optimal tax model (Sadka, 1976; Stiglitz, 1982) carries over to our setting with a graduated minimum wage.

Hence, the marginal social cost of c_1^* increases with h_1^*. As the marginal utility of c_1^* decreases with c_1^* and since we have just shown that if $h_1^* = \hat{h}_1$, then $c_1^* > \hat{c}_1$, it follows that if $h_1^* < \hat{h}_1$, then $c_1^* > \hat{c}_1$.

Finally, if $h_1^* > \hat{h}_1$, we use that the total differential of the social welfare (1) given the resource constraint (2) is

$$n_1 \left\{ \left[u'(c_1) - \frac{1}{b} \right] dc_1 + \left[\frac{af'(\ell_1)}{b} - 1 \right] dh_1 \right\}.$$

Since $u'(c_1) - 1/b > 0$ and $af'(\ell)/b < 1$, the graduated minimum wage would decrease social welfare if $c_1^* < \hat{c}_1$ in addition to $h_1^* > \hat{h}_1$. Therefore, it must be the case that if $h_1^* > \hat{h}_1$, then $c_1^* > \hat{c}_1$.

Consequently, it can be concluded that $c_1^* > \hat{c}_1$. ■

The introduction of a graduated minimum wage increases the pre-tax income of low-productivity workers, thereby making it harder for high-productivity workers to mimic low-productivity workers' income. This loosens the incentive-compatibility constraint of the high-productivity workers, which allows the government to increase the after-tax income, or consumption, of the low-productivity workers compared to the optimum with only an optimal nonlinear income tax.

The proposition guarantees that a graduated minimum wage increases social welfare. After all, the government could impose a trivial graduated minimum wage that leaves the equilibrium allocation unchanged compared to the allocation with only optimal taxes. Indeed, this is precisely the scenario with a constant minimum wage, which is always optimally nonbinding in the presence of optimal income taxation. The fact that the introduction of a graduated minimum wage leads to a different allocation means that it is both nontrivial and binding, implying that it must increase social welfare.

In essence, by loosening the incentive-compatibility constraint of the high-productivity workers, the introduction of a graduated minimum wage reduces the marginal social cost of the low-productivity workers' consumption, c_1^*. As a consequence, a graduated minimum wage allows the government to raise c_1^*. However, since the high-productivity workers' modified incentive-compatibility constraint still binds, the graduated minimum wage cannot raise c_1^* so much that it reaches the high-productivity workers' consumption and becomes first-best. Thus, the graduated minimum wage mitigates, but does not completely eliminate, the social-welfare loss that stems from the inability of the income-tax system to raise low-productivity workers' consumption to its first-best level.

The introduction of the optimal graduated minimum wage may either increase or decrease low-productivity workers' hours. On the one hand, by increasing the low-productivity workers' income for unchanged working hours, the graduated minimum wage makes it less attractive for the high-productivity workers to mimic their income. This reduces the government's need to resort to increasing the working hours of low-productivity workers as a means of loosening the high-productivity workers' incentive-compatibility constraint. On the other hand, the graduated minimum wage boosts the gain of the low-productivity workers' pre-tax income associated with an increase in their working hours. This increases the scope for further loosening of the high-productivity workers incentive-compatibility constraint through an increase in low-productivity workers' working hours. As a result of these opposing forces, the graduated minimum wage has an ambiguous effect on the low-productivity workers' hours.

That the working hours of low-productivity workers may decrease implies that the optimal graduated minimum wage, while strictly increasing social welfare, does not necessarily provide a Pareto improvement. Since the graduated minimum wage does not change the high-productivity workers' consumption and increases the low-productivity workers' consumption, total consumption increases. Therefore, (at least) in those cases where low-productivity workers' hours decrease, high-productivity workers' hours must increase and their utilities necessarily decrease. That is, social welfare increases even though high-productivity workers are made worse off.

4 The Optimal Graduated Minimum Wage

In Propositions 2-4 which follow, we determine how the characteristics of the economy affect the optimal graduated minimum wage.[8] The fact that the minimum-wage constraint (8) binds implies that $m(\ell_1^*) = af(\ell_1^*)/\ell_1^*$. In other words, the optimal realized minimum wage is equal to the low-productivity workers' average product and, therefore, due to the low-productivity workers' diminishing marginal product, inversely related to their optimal working hours. The intuition is that the government, for whichever ℓ_1^* it chooses, seeks to maximize $m(\ell_1^*)$ as a higher realized minimum wage means a looser incentive-compatibility constraint for high-productivity workers. However, due to the minimum-wage constraint, the most a firm would ever be willing to pay these low-productivity workers is their average product. Anything higher would induce the firm not to hire these workers at all. Thus, for any given ℓ_1^*, the highest realized minimum wage achievable by a graduated minimum wage is equal to the low-productivity workers' average product, and therefore the one the government chooses.

Having established the equality between the optimal realized minimum wage and the low-productivity workers' average product, a fact which plays a key role in the logic underlying the propositions in this section, we are ready to present the results. We begin with Proposition 2, which relates the revenue needs of the government to the optimal realized minimum wage.

Proposition 2 *The optimal realized minimum wage decreases with the government's revenue needs; that is, $dm(\ell_1^*)/dR < 0$.*

[8] The proofs of Propositions 2-4 are in the Appendix.

The higher the government's revenue needs, the less output is available for the low-productivity workers' consumption for any given working hours. As the marginal utility of low-productivity workers' consumption decreases with an increase in their consumption, the social gain from having them work more will be higher.[9] Accordingly, the low-productivity workers' hours increase with the government's revenue needs. Since increased working hours of low-productivity workers lower their average product, it also lowers the optimal realized minimum wage.

We now examine the impact of the number of workers of each type on the optimal realized minimum wage.

Proposition 3 *The optimal realized minimum wage:*

1. *decreases with the number of low-productivity workers; that is,* $dm(\ell_1^*)/dn_1 < 0$*;*
2. *increases with the number of high-productivity workers; that is,* $dm(\ell_1^*)/dn_2 > 0$*.*

The first part of the proposition shows that the optimal realized minimum wage is inversely related to the number of low-productivity workers. The more low-productivity workers there are, the lower is their per-capita consumption for a fixed total number of low-productivity worker hours. Since workers have decreasing marginal utility from consumption, the social gain from additional working hours for low-productivity workers increases. Similarly to the logic underlying Proposition 2, this will increase the socially optimal number of total working hours for low-productivity workers (although each individual worker's hours may be lower). In the social optimum, therefore, total hours of low-productivity workers within each firm is greater when the number of low-productivity workers is greater. This lowers their average product and hence the optimal realized minimum wage.

[9] Of course, the low-productivity workers may benefit from higher public expenditues if used for public goods that primarily benefit the low-productivity workers.

The second part of the proposition shows that the number of high-productivity workers affects the optimal realized minimum wage positively. Since high-productivity workers produce more than they consume, an increase in their numbers facilitates the transfer of consumption to low-productivity workers. Again, since workers have decreasing marginal utility from consumption, this transfer reduces the marginal social gain from additional work of the low-productivity workers. Therefore, low-productivity workers will work less, which raises their average product and hence the optimal realized minimum wage.

In our final proposition, we establish the relationship between each type of worker's productivity and the optimal realized minimum wage.

Proposition 4 *The optimal realized minimum wage:*

1. *increases with the low-productivity workers' productivity; that is,* $dm(\ell_1^*)/da > 0$*;*
2. *increases with the high-productivity workers' productivity; that is,* $dm(\ell_1^*)/db > 0$*.*

When wages are competitively determined, then it is, of course, unsurprising that an increase in workers' productivity leads to an increase in their wages. However, because here the wage paid to low-productivity workers is not competitively determined, it is far less obvious that the optimal realized minimum wage should increase with the productivity of low-productivity workers. Nevertheless, the first part of the proposition confirms that this is indeed the case.

The intuition can be summarized as follows: An increase in the productivity of low-productivity workers increases their consumption because it reduces the social cost of their consumption, both by lowering the disutility associated with the production of additional output and by loosening the high-productivity workers' incentive-compatibility constraint. At the same time, as low-productivity workers' consumption increases (and their marginal utility from consumption decreases), a greater marginal product of low-productivity labor is required to make additional working hours socially beneficial. Therefore, since the increase in productivity leads to an increase in consumption for low-productivity workers, it must also lead to an increase in their marginal product in the social optimum. This then implies a higher average product and consequently a greater optimal realized minimum wage.

We now turn to the logic underlying the second part of the proposition, that the optimal realized minimum wage is positively related to the productivity of high-productivity workers. The higher the productivity of high-productivity workers, the less attractive are additional low-productivity working hours relative to high productivity working hours from a social-welfare point of view. This will tend to decrease the working hours of low-productivity workers, thereby raising their average product and, as a result, the optimal realized minimum wage.

5 Conclusion

This paper is the first to study the properties of a graduated minimum wage introduced to increase social welfare in a competitive environment. Our starting point is the well-established theoretical insight that a constant minimum wage cannot increase welfare beyond what can be obtained by an optimal income tax alone. The government's dilemma is that although it would like to redistribute resources from high-productivity workers to low-productivity workers, it is limited in its ability to do so by an incentive-compatibility constraint. Specifically, the government's redistribution scheme cannot be so generous that high-productivity workers find it preferable to mimic low-productivity workers by earning the same income and working fewer hours. However, unlike a constant minimum wage, a graduated minimum wage, by increasing the pre-tax income of low-productivity workers, makes it more difficult for high-productivity workers to mimic low-productivity workers since the former would need to work more hours to earn the income of the latter. This allows the government to further redistribute to the low-productivity workers by increasing their after-tax income by more than it could have otherwise.

An important result is, therefore, that when the government's toolbox of available policies is expanded to include a nonconstant minimum wage, welfare can be improved beyond what is achievable with only income taxes and a constant minimum wage. And why should the government not have such a policy tool available to it? After all, if the government can impose nonlinear income taxes in the interest of social welfare, why not a nonconstant minimum wage as well? It seems rather arbitrary to allow one but prohibit the other.

The concept of a graduated minimum wage is not only of theoretical interest. As a matter of fact, there is some precedent for a nonconstant minimum wage in practice. For instance, in the United States, 13 states impose size-dependent minimum wages on firms, and Colombia, Honduras, and Panama have legislated multi-bracket minimum-wage schedules which make the minimum wage depend on the firm size.

What are the properties of an optimal size-dependent, or as we refer to it, graduated, minimum wage? In this paper we have shown that a welfare-maximizing graduated minimum wage increases the low-productivity workers' consumption above its level with an optimal income tax alone, bringing it closer to the first-best. We have also shown that the realized minimum wage in a social-welfare optimum depends on important economy characteristics. On the one hand, the optimal realized minimum wage decreases with government's revenue needs and with the number of low-productivity workers. On the other hand, it increases with the number of high-productivity workers and the productivities of the different types of workers.

We see this paper as a first step toward characterizing the optimal graduated minimum wage and have therefore abstracted from some potentially important considerations. In particular, we see the inclusion of additional worker and firm heterogeneity into our framework as important avenues for future research that will provide a fuller characterization of the optimal graduated minimum wage. Since these additional considerations should not negate the basic insight that a graduated minimum wage is a useful policy instrument that governments can use to improve social welfare, we hope that the framework developed in this paper can serve as a basis for further analysis of the merits of a graduated minimum wage.

Appendix

We prove Propositions 2-4 by differentiating Condition (9) and the binding constraints (2), (7), and (8). Let

$$
\begin{aligned}
A &\equiv [(n_1 + 2n_2)af'(\ell_1^*) - n_2 b]^2, \\
B &\equiv -\frac{m(\ell_1^*) - af'(\ell_1^*)}{(n_1+n_2)h_1^* \left\{n_1(n_1+n_2)\left[n_1 + n_2 bU'(c_1^*)\right] af''(\ell_1^*) + af'(\ell_1^*)AU''(c_1^*)\right\}},
\end{aligned}
$$

where $m(\ell_1^*) - af'(\ell_1^*) > 0$, $af''(\ell_1^*) < 0$, and $U''(c_1^*) < 0$ imply that $B > 0$.

Differentiating with respect to R yields

$$\frac{dm(\ell_1^*)}{dR} = ABU''(c_1^*).$$

It follows that $dm(\ell_1^*)/dR < 0$, which proves Proposition 2.

Differentiating with respect to n_1 yields

$$\frac{dm(\ell_1^*)}{dn_1} = -\frac{B}{n_1 b}\left\{n_1 n_2 [b - af'(\ell_1^*)]^2 \left[n_1 + n_2 bU'(c_1^*)\right] - [n_1 c_1^* + n_2 m(\ell_1^*)h_1^*] bAU''(c_1^*)\right\}.$$

It follows that $dm(\ell_1^*)/dn_1 < 0$, which proves part 1 of Proposition 3.

Differentiating with respect to n_2 yields

$$\frac{dm(\ell_1^*)}{dn_2} = \frac{B}{b}\left\{n_1 [b - af'(\ell_1^*)]^2 \left[n_1 + n_2 bU'(c_1^*)\right] - b(bh_2^* - c_2^*)AU''(c_1^*)\right\}.$$

Since there is redistribution away from the high-productivity workers, in a social-welfare optimum $bh_2^* > c_2^*$. Hence, $dm(\ell_1^*)/dn_2 > 0$, which proves part 2 of Proposition 3.

Differentiating with respect to a yields

$$\frac{dm(\ell_1^*)}{da} = -B[n_2 m(\ell_1^*)h_1^* + af(\ell_1^*)]AU''(c_1^*).$$

It follows that $dm(\ell_1^*)/da > 0$, which proves part 1 of Proposition 4.

Differentiating with respect to b yields

$$\frac{dm(\ell_1^*)}{db} = \frac{n_2 B}{b^2}\left\{[b - af'(\ell_1^*)]\left[n_1 b + (n_1 + 2n_2)af'(\ell_1^*)\right]\left[n_1 + n_2 bU'(c_1^*)\right] - bAU''(c_1^*)[bh_2^* - m(\ell_1^*)h_1^*]\right\}.$$

The term in the braces is positive since $b > af'(\ell_1^*)$ and the high-productivity workers' modified incentive-compatibility constraint implies that $bh_2^* - m(\ell_1^*)h_1^* > 0$. Consequently, $dm(\ell_1^*)/db > 0$, which proves part 2 of Proposition 4. ■

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